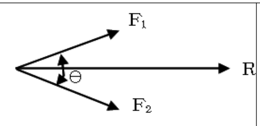
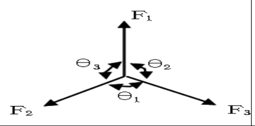
재료역학

두 힘의 합성

	$R=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

라미의 정리

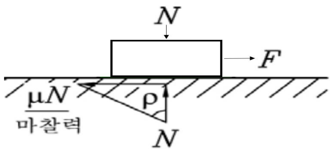
	$\dfrac{F_1}{\sin\theta_1}=\dfrac{F_2}{\sin\theta_2}=\dfrac{F_3}{\sin\theta_3}$

힘과 모멘트 평형 : 정역학적 평형상태 방정식

$\sum F_{x,y,z}=0,\ \sum M_{x,y,z}=0$

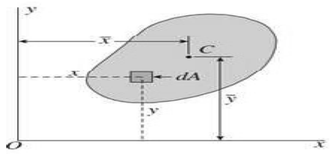
자유물체도(F.B.D) : 물체에 작용하는 모든 힘을 표시

마찰력 : 수직력의 함수

	$F_f=\mu N$ $\tan\rho=\dfrac{\mu N}{N}=\mu$(마찰계수)

단면 1차 모멘트 : 단면×거리

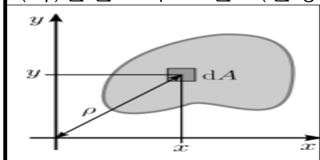
면적에 대한 임의 축에 대한 모멘트는 미소면적에 대한 모멘트 합과 같다.

	$Q_x=\bar{y}A=\int_A y\,dA$ $Q_y=\bar{x}A=\int_A x\,dA$

도심

$\bar{x}=\dfrac{\sum A_i x_i}{\sum A_i}=\dfrac{A_1x_1+A_2x_2}{A_1+A_2},\ \bar{y}=\dfrac{\sum A_i y_i}{\sum A_i}=\dfrac{A_1y_1+A_2y_2}{A_1+A_2}$

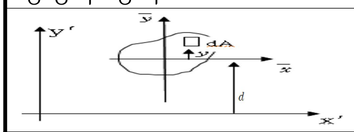
(극)단면 2차 모멘트(관성 모멘트) : 단면×거리^2

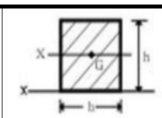
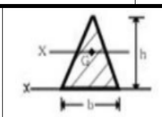
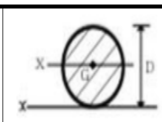
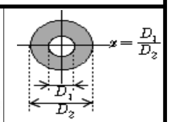
	$I_x=\int_A y^2dA,\ I_y=\int_A x^2dA$ $I_P=\int r^2dA=\int(x^2+y^2)dA$ $=I_x+I_y$

(극)단면계수

$Z=\dfrac{I_x}{e},\ Z_P=\dfrac{I_P}{e}$, e : 도심에서 최외단까지의 거리

평행축 정리

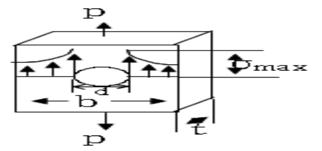
	$I_{x'}=I_x+Ad^2$ (d : 두 축 사이의 거리) d

구분				 $x=\frac{D_1}{D_2}$
$I_{x'}$	$\dfrac{bh^3}{3}$	$\dfrac{bh^3}{12},\dfrac{bh^3}{4}$		
I_x	$\dfrac{bh^3}{12}$	$\dfrac{bh^3}{36}$	$\dfrac{\pi d^4}{64}$	$\dfrac{\pi d_2^4}{64}(1-x^4)$
I_P			$\dfrac{\pi d^4}{32}$	$\dfrac{\pi d_2^4}{32}(1-x^4)$
Z	$\dfrac{bh^2}{6}$	$\dfrac{bh^2}{12},\dfrac{bh^2}{24}$	$\dfrac{\pi d^3}{32}$	$\dfrac{\pi d_2^3}{32}(1-x^4)$
Z_P			$\dfrac{\pi d^3}{16}$	$\dfrac{\pi d_2^3}{16}(1-x^4)$

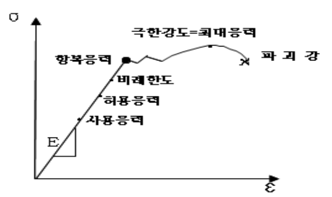
응력과 변형률

수직응력 $\sigma=\dfrac{P}{A}=E\epsilon$	전단응력 $\tau=\dfrac{P}{A}=G\gamma$
종변형률 $\epsilon=\dfrac{\lambda\ (=\Delta\ell)}{\ell}$ $\lambda=\dfrac{P\ell}{AE}$	전단변형률 $\gamma=\tan\theta=\dfrac{\lambda_s}{\ell}$ $\lambda_s=\dfrac{P\ell}{AG}$
횡변형률 $\epsilon'=\dfrac{\delta\ (=\Delta d)}{d}$	탄성계수 관계식 $E=2G(1+\mu)=3K(1-2\mu)$ E: 종탄성계수 G: 횡탄성계수 K: 체적탄성계수 m: 프와송 수
포와송 비 $\mu=\dfrac{\epsilon'}{\epsilon}=\dfrac{1}{m}$	
단면변형률 $\epsilon_A=\dfrac{\Delta A}{A}=2\mu\epsilon$	
체적변형률 $\epsilon_V=\dfrac{\Delta V}{V}=\epsilon(1-2\mu)$	

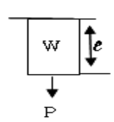
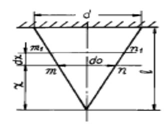
응력 집중

	최대응력 $\sigma_{\max}=\alpha_k\sigma_{av}$ 평균(공칭)응력 $\sigma_{av}=\dfrac{P}{A}=\dfrac{P}{(b-d)t}$

응력-변형률 선도

	σ_w : 사용응력 σ_a : 허용응력 σ_u : 극한강도(최대응력) S: 안전율 $\sigma_w\le\sigma_a=\dfrac{\sigma_u}{S},\ S=\dfrac{\sigma_u}{\sigma_a}$

자중을 고려한 응력 및 변형량(균일 단면봉)

	$\sigma=\dfrac{P}{A}+\gamma\ell$ $\lambda=\dfrac{P\ell}{EA}+\dfrac{\gamma\ell^2}{2E}$		$\sigma=\dfrac{\gamma\ell}{3}$ $\lambda=\dfrac{\gamma\ell^2}{6E}$

열응력

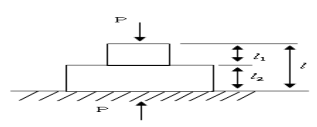
$\sigma=E\epsilon=E\alpha\cdot\Delta T$

$\epsilon=\alpha\cdot\Delta T$ (α : 열팽창계수, 1/℃)

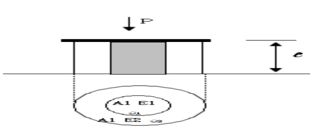
$\lambda=\epsilon\ell=\alpha\Delta T\ell$

$P=EA\alpha\cdot\Delta T$

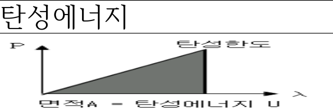
조합된 부재(직렬조합)

	$\sigma_1=\dfrac{P}{A_1},\ \sigma_2=\dfrac{P}{A_2}$ $\lambda=\lambda_1+\lambda_2=\dfrac{P\ell_1}{E_1A_1}+\dfrac{P\ell_2}{E_2A_2}$

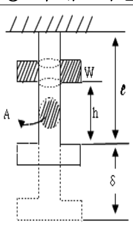
조합된 부재(병렬조합)

	$\sigma_1=\dfrac{PE_1}{E_1A_1+E_2A_2}$ $\sigma_2=\dfrac{PE_2}{E_1A_1+E_2A_2}$

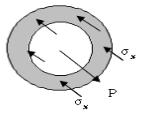
탄성에너지

	$U=\dfrac{P\lambda}{2}=\dfrac{P^2\ell}{2EA}=\dfrac{\sigma^2V}{2E}$

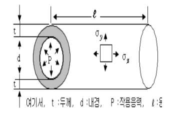
충격에 의한 응력과 변형량

	$\sigma_0=\dfrac{W}{A},\ \lambda_0=\dfrac{W\ell}{EA}$ $\sigma=\sigma_0\left(1+\sqrt{1+\dfrac{2h}{\lambda_0}}\right),\ \lambda=\lambda_0\left(1+\sqrt{1+\dfrac{2h}{\lambda_0}}\right)$ ※ $h=0$일 경우 $\sigma=2\sigma_0,\ \lambda=2\lambda_0$

내압을 받는 얇은 용기



$$\sigma_s = \frac{P_i d}{4t}$$

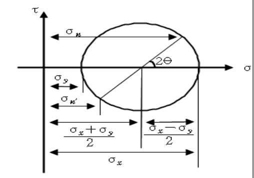


$$\sigma_h = \frac{P_i d}{2t}$$

얇은 회전체의 응력(풀리, 림, 플라이휠 등)

$$\sigma = \frac{\gamma V^2}{g}$$

모어의 응력원

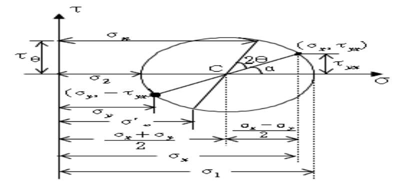


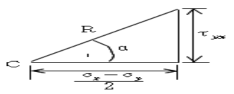
① σ, τ축을 잡는다.

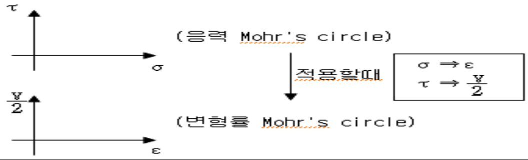
② $\sigma_x, \sigma_y, \tau_{xy}$표시

③점들이 만족하는 지름을 긋는다.

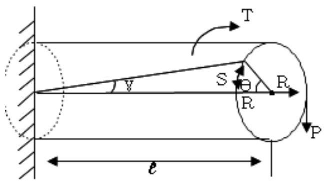
④ 2θ를 긋는다.







비틀림



$$\tau = \frac{Gr\theta}{\ell} = \frac{T}{Z_P}$$

$$\theta = \frac{T\ell}{GI_P}[rad]$$

$$U = \frac{1}{2}T\theta$$

바하(Bach)의 축 공식

축의 재질 : 연강, $G = 0.83 \times 10^6 kg/cm^2$,

$1m$당 비틀림각은 $\frac{1}{4}^\circ (= 0.25^\circ)$

보의 분포하중, 전단력, 굽힘모멘트 관계

$$w = \frac{dF}{dx} = \frac{d^2M}{dx^2}(S.F.D\text{기울기})$$

$$V = \frac{dM}{dx}(B.M.D\text{기울기})$$

S.F.D 및 B.M.D 그리는 순서

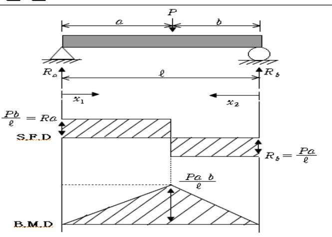
① 반력 및 반력 모멘트를 구한다.

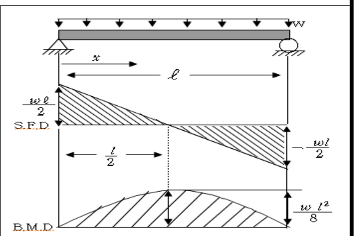
② 하중의 연속유무에 따라 구간을 나눈다,

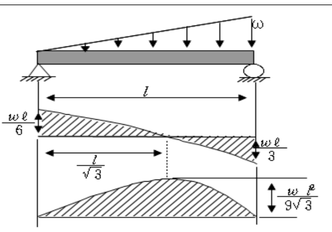
③ 구간을 나누어서 각 구간의 자유물체도를 그린다.

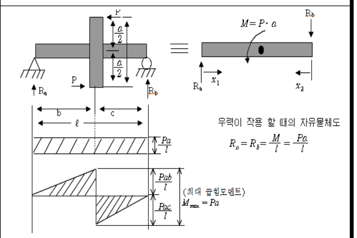
④ 자유물체도에서의 V_x, M_x 구한 후 선도를 그린다.

단순보

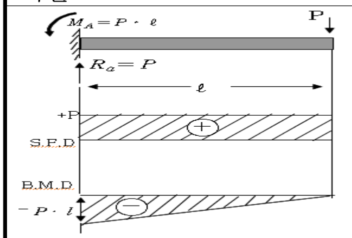


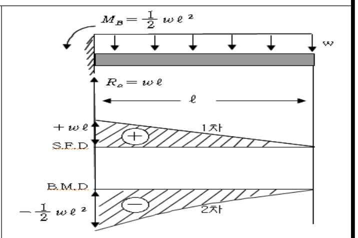


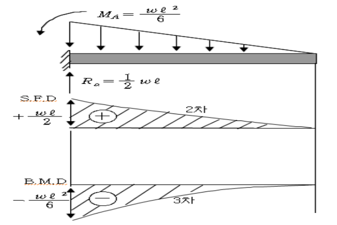


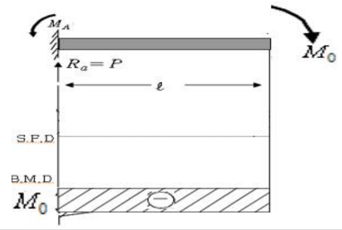


외팔보









보의 굽힘응력

$$\sigma = E\frac{y}{\rho}, \frac{1}{\rho} = \frac{M}{EI}, \frac{E}{\rho} = \frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma_y = \frac{M}{I}y, \sigma_b = \frac{M}{Z} \ (\rho: \text{곡률반경}, \frac{1}{\rho}: \text{곡률})$$

보의 전단응력

$$\tau = \frac{VQ}{Ib}$$

(V: 전단력, Q: 도심 축으로부터 반절단면의 1차 모멘트)

사각형 단면 $\tau_{\max} = \frac{3V}{2A}$, 원형 단면 $\tau_{\max} = \frac{4V}{3A}$

굽힘 모멘트와 비틀림 모멘트가 동시에 작용할 때

상당비틀림모멘트

$$T_e = \sqrt{M^2 + T^2}$$

상당굽힘모멘트

$$M_e = \frac{1}{2}(M + \sqrt{M^2 + T^2}) = \frac{1}{2}(M + T_e)$$

$$\tau_{\max} = \frac{T_e}{Z_P}, \sigma_{\max} = \frac{M_e}{Z}$$

보의 처짐

미분방정식

$$EIy'''' = \frac{dV_x}{dx} = w_x$$

$$EIy''' = \frac{dM_x}{dx} = V_x$$

$$EIy'' = M_x$$

$$EIy' = \int M_x dx \left(y' = \frac{dy}{dx} = \text{처짐각} = \theta\right)$$

$$EIy = \iint M_x dx dx \ (y = \text{처짐량} = \delta)$$

보의 처짐 해석방법

① 미분방정식 활용

② 면적 모멘트법

③ 중첩법

④ 카스틸리아노 정리(탄성 에너지법)

면적 모멘트법

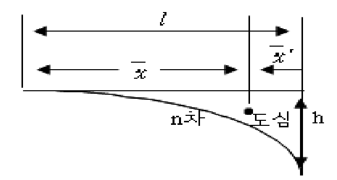
$$\theta = \frac{A_M}{EI}$$

$$\delta = \frac{A_M}{EI}\bar{x}$$

A_M : $B.M.D$의 면적

$\bar{x}$: 처짐을 구하고자하는 그 위치로부터 A_M의 도심까지의 거리ㅏ

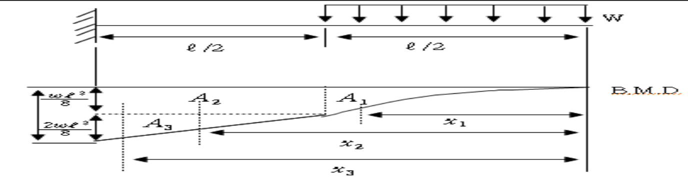
면적과 도심 구하는 일반식



$$A = \frac{hl}{n+1}$$

$$\bar{x} = \frac{(n+1)l}{n+2}$$

$$\bar{x}' = \frac{l}{n+2}$$

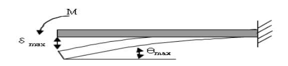
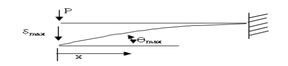
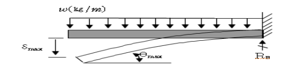
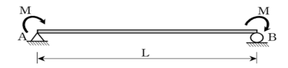
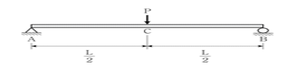
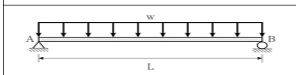


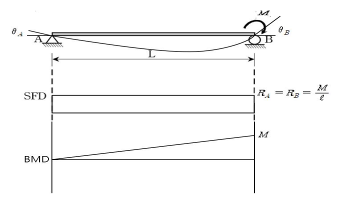
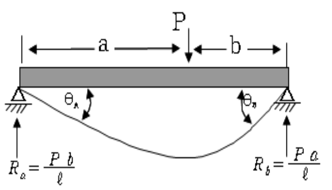
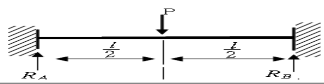
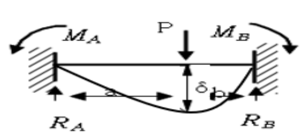
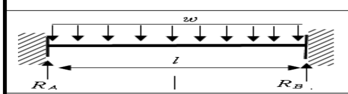
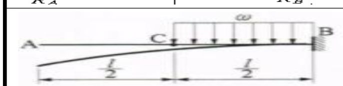
$$\theta = \frac{A_M}{EI} = \frac{A_1 + A_2 + A_3}{EI}$$

$$\delta = \frac{A_M}{EI}\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{EI}$$

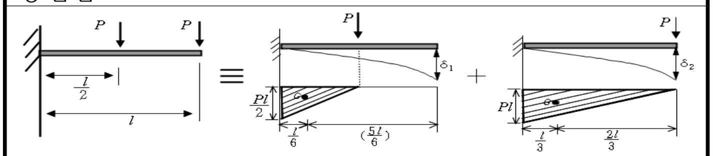
카스틸리아노 정리(탄성 에너지법)

$$U = \int_0^{\ell} \frac{M_x^2\,dx}{2EI},\ \theta = \frac{\partial U}{\partial M},\ \delta = \frac{\partial U}{\partial P}$$

구분		θ	δ
외팔		$\frac{M\ell^1}{1EI}$	$\frac{M\ell^2}{2EI}$
		$\frac{P\ell^2}{2EI}$	$\frac{P\ell^3}{3EI}$
		$\frac{w\ell^3}{6EI}$	$\frac{w\ell^4}{8EI}$
단순		$\frac{M\ell^1}{2EI}$	$\frac{M\ell^2}{8EI}$
		$\frac{P\ell^2}{16EI}$	$\frac{P\ell^3}{48EI}$
		$\frac{w\ell^3}{24EI}$	$\frac{5w\ell^4}{384EI}$

	$\theta_A = \frac{M\ell}{6EI},\ \theta_B = \frac{M\ell}{3EI}$ $x = \frac{\ell}{\sqrt{3}},\ \delta_{\max} = \frac{M\ell^2}{9\sqrt{3}EI}$ 중앙점의 처짐$\delta = \frac{M\ell^2}{16EI}$
	$\theta_A = \frac{Pab(\ell+b)}{6\ \ell EI}$ $\theta_B = \frac{Pab(\ell+a)}{6\ \ell EI}$ $\delta = \frac{Pa^2b^2}{3\ell EI}$
	$\theta = \frac{P\ell^2}{64EI},\ \delta = \frac{P\ell^3}{192EI}$
	$M_A = \frac{Pab^2}{\ell^2},\ M_B = \frac{Pa^2b}{\ell^2}$ $\delta = \frac{Pa^2b^2}{3\ell^3EI}$
	$\theta = \frac{w\ell^3}{125EI},\ \delta = \frac{w\ell^4}{384EI}$
	$\theta = \frac{w\ell^3}{48EI},\ \delta = \frac{7w\ell^4}{384EI}$

중첩법

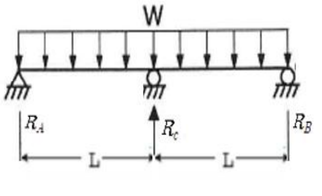
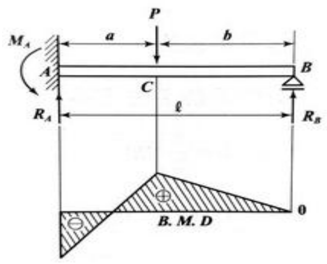
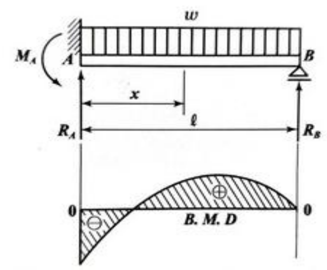
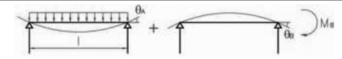
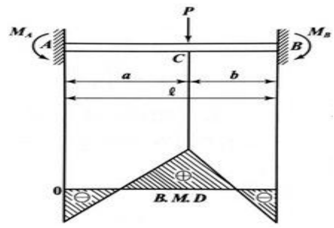
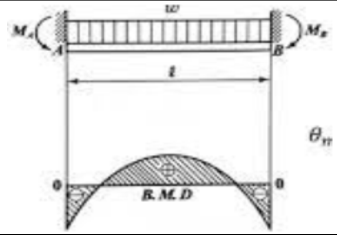


$$\delta = \delta_1 + \delta_2,\ \theta = \theta_1 + \theta_2$$

$$\delta_1 = \frac{5P_1\ell^3}{48EI},\ \theta_1 = \frac{P_1\ell^2}{8EI}$$

$$\delta_2 = \frac{P_2\ell^3}{3EI},\ \theta_2 = \frac{P_2\ell^2}{2EI}$$

부정정보

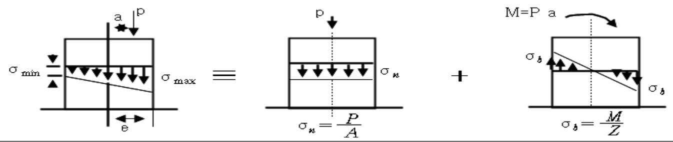
	 $\delta_A = \delta_B$ $\frac{5w(2\ell)^4}{384EI} = \frac{R_c(2\ell)^3}{48EI}$ $R_c = \frac{5}{4}w\ell,\ R_A = R_B = \frac{3}{8}w\ell$
	 $a = b,\ \theta_A + \theta_B = 0$ $\frac{P\ell^2}{16EI} = \frac{M_A\ell}{3EI}$ $M_A = \frac{3P\ell}{16}$ $R_A = \frac{11}{16}P,\ R_B = \frac{5}{16}P$
	 $a = b,\ \theta_A + \theta_B = 0$ $\frac{w\ell^3}{24EI} = \frac{M_A\ell}{3EI}$ $M_A = \frac{w\ell^2}{8}$ $R_A = \frac{5}{8}w\ell,\ R_B = \frac{3}{8}w\ell$
	 $a = b,\ \theta_A + \theta_B = 0$ $\frac{P\ell^2}{16EI} = \frac{M\ell}{2EI}$ $M = \frac{P\ell}{8},\ R_A = R_B = \frac{P}{2}$
	 $\theta_A + \theta_B = 0$ $\frac{w\ell^3}{24EI} = \frac{M\ell}{3EI} + \frac{M\ell}{6EI} = \frac{M\ell}{2EI}$ $M = \frac{w\ell^2}{12},\ R_A = R_B = \frac{w\ell}{2}$

기둥

핵심반경 : $a = \frac{K^2}{e}$, 세장비 : $\lambda = \frac{\ell}{K}$

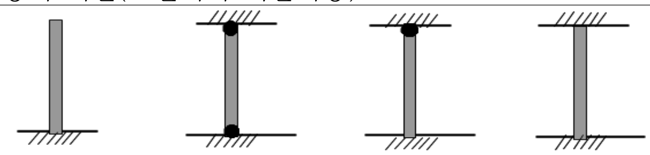
회전반경 : $K = \sqrt{\frac{I}{A}}$, 기둥의 길이 : ℓ

편심하중을 받는 단주



$$\sigma_{\max,\min} = \frac{P}{A} \pm \frac{M}{Z}$$

기둥의 좌굴(오일러의 좌굴하중)



<일단고정타단자유>	<양단회전>	<일단고정타단회전>	<양단고정>
n =1/4	n=1	n=2	n=4

좌굴하중	좌굴응력
$P_{cr} = n\pi^2\frac{EI}{\ell^2}$	$\sigma_{cr} = \frac{P_{cr}}{A} = n\pi^2\frac{E}{\ell^2}\frac{I}{A} = n\pi^2\frac{E}{\ell^2}K^2 = n\pi^2\frac{E}{\lambda^2}$