< 정답 및 해설 >

<정답>

문항	1	2	3	4	5	6	7	8	9	10	11	12	13
정답	④	③	④	②	①	①	②	③	①	③	②	①	④
문항	14	15	16	17	18	19	20	21	22	23	24	25	
정답	④	③	①	②	①	③	②	④	②	③	④	②	

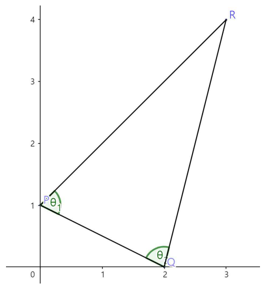
<해설>

1. 역함수 $g(x)=\sqrt{3-x}$. 따라서 $g\circ g(x)=\sqrt{3-\sqrt{3-x}}$

   정의역: $3-\sqrt{3-x}\ \geq 0$으로부터 $-6\leq x\leq 3$

2. $\cos\left(\frac{4\pi}{3}\right)=-\frac{1}{2}\ \therefore\ \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

3. $P(0,1),Q(2,0),R(3,4)$



$\overrightarrow{PR}$과 $\overrightarrow{PQ}$ 사이 각θ_1, $\cos\theta_1=\frac{\langle 3,3\rangle\cdot\langle 2,-1\rangle}{\sqrt{90}}=\frac{1}{\sqrt{10}}$

$\overrightarrow{QP}$과 $\overrightarrow{QR}$ 사이 각θ_2, $\cos\theta_2=\frac{\langle -2,1\rangle\cdot\langle 1,4\rangle}{\sqrt{85}}=\frac{2}{\sqrt{85}}$

$\frac{2}{\sqrt{85}}<\frac{1}{\sqrt{10}}$이므로 $\theta_2>\theta_1$이고 $\theta_2=\cos^{-1}\left(\frac{2\sqrt{85}}{85}\right)$

4. $\lim_{x\to\infty}\left(1+\frac{e}{x}\right)^x=\lim_{x\to\infty}\left\{\left(1+\frac{e}{x}\right)^{x/e}\right\}^e=e^e$

5. $\frac{d}{dx}\int_1^{e^x}(1+\ln t)\,dt=(1+\ln e^x)e^x=(1+x)e^x$

6. $\int_3^8\frac{\sqrt{1+x}}{x}dx=\int_2^3\frac{2t^2}{t^2-1}dt=\int_2^3 2+\frac{2}{t^2-1}dt=[2t+\ln(t-1)-\ln(t+1)]_2^3=2+\ln\frac{3}{2}$

7. $\displaystyle\int_0^{\pi/2}\frac{\sin 2x}{2+\cos x}\,dx=\int_0^{\pi/2}\frac{2\sin x\cos x}{2+\cos x}\,dx$

$$=\int_0^1 2-\frac{4}{2+t}\,dt=[2t-4\ln(2+t)]_0^1 \;=2-4\ln\frac{3}{2}$$

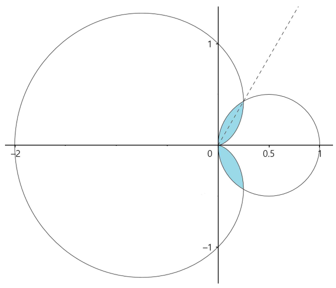
8. 접선의 방정식 $\quad y=y(1)+\dfrac{y'(1)}{x'(1)}(x-x(1))=3+\dfrac{6}{3}(x-0)\qquad\qquad \therefore\; y=2x+3$

9. 넓이$\displaystyle =2\left(\int_{\pi/3}^{\pi/2}\frac{1}{2}\cos^2\theta\,d\theta+\int_0^{\pi/3}\frac{1}{2}(1-\cos\theta)^2\,d\theta\right)$

$$=\int_{\pi/3}^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta+\int_0^{\pi/3}1-2\cos\theta+\frac{1+\cos 2\theta}{2}\,d\theta$$

$$=\int_0^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta+\int_0^{\pi/3}1-2\cos\theta\,d\theta$$

$$=\frac{1}{2}\cdot\frac{\pi}{2}+\frac{\pi}{3}-\sqrt{3}\;=\frac{7\pi}{12}-\sqrt{3}$$



10. $\displaystyle\vec{u}=\frac{\langle 2,3\rangle}{\sqrt{13}},\; D_{\vec{u}}f(1,1)=\nabla f(1,1)\cdot\vec{u}=\left(3^{xy}\ln 3\langle y,x\rangle\right)_{(1,1)}\cdot\vec{u}$

$$=3\ln 3\langle 1,1\rangle\cdot\frac{\langle 2,3\rangle}{\sqrt{13}}=\frac{15\ln 3}{\sqrt{13}}$$

11. $\displaystyle\frac{\partial z}{\partial x}=-\frac{e^{xyz}yz+\frac{1}{x}}{e^{xyz}xy+\frac{1}{z}}=-\frac{\frac{e^{xyz}xyz+1}{x}}{\frac{e^{xyz}xyz+1}{z}}=-\frac{z}{x}$

$$\frac{\partial z}{\partial y}=-\frac{e^{xyz}xz+\frac{1}{x}}{e^{xyz}xy+\frac{1}{z}}=-\frac{\frac{e^{xyz}xyz+1}{y}}{\frac{e^{xyz}xyz+1}{z}}=-\frac{z}{y}\qquad\qquad \therefore\left(\frac{\partial z}{\partial x}\right)\cdot\left(\frac{\partial z}{\partial y}\right)=\frac{z^2}{xy}$$

12. 접평면: $\displaystyle z=f(1,1)+\left(\tan^{-1}\left(\frac{y}{x}\right)\right)_x(1,1)(x-1)+\left(\tan^{-1}\left(\frac{y}{x}\right)\right)_y(1,1)(y-1)$

$$=\frac{\pi}{4}+\left(-\frac{1}{2}\right)(x-1)+\frac{1}{2}(y-1)=\frac{\pi}{4}-\frac{1}{2}x+\frac{1}{2}y\qquad\qquad \therefore x-y+2z=\frac{\pi}{2}$$

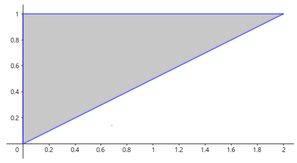
13. $u_x=yf'(x-y),\; u_{xx}=yf''(x-y)$

$u_y=f(x-y)-yf'(x-y),\; u_{yy}=-f'(x-y)-f'(x-y)+yf''(x-y)$

$\therefore\; u_{xx}(1,1)-u_{yy}(1,1)=2f'(0)=6$

14. $\int_0^2 \int_{x/2}^1 x\sin(y^3-1)\,dy\,dx = \int_0^1 \int_0^{2y} x\sin(y^3-1)\,dx\,dy = \int_0^1 \frac{4y^2}{2}\sin(y^3-1)\,dy$

$= \frac{2}{3}\left[-\cos(y^3-1)\right]_0^1 = -\frac{2}{3}(1-\cos(1))$

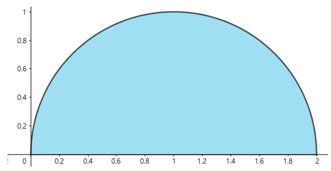


15. $0 \le x \le 2$, $0 \le y \le \sqrt{2x-x^2}$ 의 극좌표영역 $0 \le \theta \le \frac{\pi}{2}$, $0 \le r \le 2\cos\theta$

$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx = \int_0^{\pi/2} \int_0^{2\cos\theta} r^2\,dr\,d\theta$

$= \int_0^{\frac{\pi}{2}} \frac{8}{3}\cos^3\theta\,d\theta$

$= \frac{8}{3}\left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2} = \frac{16}{9}$



16. $\iiint_E 30x\,dV = \int_0^1 \int_y^2 \int_0^{1-y^2} 30x\,dz\,dx\,dy = \int_0^1 \int_y^2 30x(1-y^2)\,dx\,dy$

$= \int_0^1 15(4-y^2)(1-y^2)\,dy = 38$

17. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{1-x^2-y^2}^4 \sqrt{x^2+y^2}\,dz\,dy\,dx = \int_0^{\pi/2} \int_0^1 \int_{1-r^2}^4 r^2\,dz\,dr\,d\theta$

$= \int_0^{\pi/2} \int_0^1 r^2(4-1+r^2)\,dr\,d\theta$

$= \int_0^{\pi/2} \left[r^2 + \frac{r^5}{5}\right]_0^1 d\theta = \frac{6}{5} \cdot \frac{\pi}{2} = \frac{3\pi}{5}$

18. $(\cot x)y' + 3(1+y^2) = 0$

$\frac{1}{1+y^2}dy + 3\tan x\,dx = 0$ 적분하면 $\tan^{-1}y + 3\ln|\sec x| = C$

$\therefore\ y = \tan(C - 3\ln|\sec x|)$

19. $((1+x^2)y)' = \cos x\ \ \therefore (1+x^2)y = \sin x + C$ 따라서 $y = \frac{\sin x + C}{1+x^2}$

20. $f_x = e^y \sin x + 2x$ 에서 $f(x,y) = -e^y\cos x + x^2 + \phi(y)$

$f_y = -e^y\cos x + \phi'(y)\quad \therefore\ \phi(y) = -2y + C$

따라서 $e^y\cos x - x^2 + 2y = C$

21. $y_p = \cos x \int \frac{-\sin x \csc^2 x}{1}\, dx + \sin x \int \cos x \csc^2 x\, dx$

$= -\cos x \ln|\csc x - \cot x| - 1$

$y_g = c_1 \cos x + c_2 \sin x - \cos x \ln|\csc x - \cot x| - 1$

22. 특성 방정식은 $\lambda^2 + 2\lambda + 1 = 0$ 이므로 $\lambda = -1$ (중근)

$y_c = c_1 e^{-x} + c_2 x e^{-x}$

$y_p = \frac{1}{2} x^2 e^x$

$y = c_1 e^{-x} + c_2 x e^{-x} + \frac{1}{2} x^2 e^{-x}$

초기조건을 대입하면 $c_1 = 0,\ c_2 = 1 \quad \therefore y = (x + \frac{1}{2}x^2) e^{-x}$

23. $f(t) = \sin t - \sin t\, u(t-\pi) = \sin t + \sin(t-\pi)\, u(t-\pi)$

$\mathcal{L}\{\sin t + \sin(t-\pi)\, u(t-\pi)\} = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1}$

24. $\mathcal{L}^{-1}\left(\frac{s}{s^2+2s+5}\right) = \mathcal{L}^{-1}\left(\frac{(s+1)-1}{(s+1)^2+2^2}\right) = e^{-t}(\cos 2t - \frac{1}{2}\sin 2t)$ 이므로

제2 이동정리에 의해

$f(t) = \mathcal{L}^{-1}\left(\frac{s}{s^2+2s+5} e^{-\frac{\pi}{2}s}\right) = e^{-(t-\frac{\pi}{2})}(\cos 2(t-\frac{\pi}{2}) - \frac{1}{2}\sin 2(t-\frac{\pi}{2}))\, u(t-\frac{\pi}{2}).$

$\therefore\ f(\pi) = -e^{-\frac{\pi}{2}}$

25. 두 번째 해 $y_2 = y_1 \int \frac{e^{-\int p(x)\,dx}}{(y_1)^2}\, dx$

$\int \frac{e^{-\int p(x)\,dx}}{(y_1)^2}\, dx = \int \frac{e^{-\int \frac{4x}{1+2x} dx}}{e^{-4x}}\, dx = \int \frac{e^{-\int 2 - \frac{2}{1+2x} dx}}{e^{-4x}}\, dx$

$= \int \frac{e^{-2x + \ln(1+2x)}}{e^{-4x}}\, dx = \int e^{2x}(1+2x)\, dx = xe^{2x}$

$\therefore\ y_2 = e^{-2x}\, x e^{2x} = x$