< 정답 및 해설 >

<정답>

문항	1	2	3	4	5	6	7	8	9	10	11	12	13
정답	③	①	①	②	③	②	①	③	②	④	①	②	④
문항	14	15	16	17	18	19	20	21	22	23	24	25	
정답	②	④	①	④	②	②	④	③	③	③	④	①	

<해설>

1. $\lim\limits_{x\to0}\dfrac{e^{3x}-1-3x}{x^2}=\lim\limits_{x\to0}\dfrac{3e^{3x}-3}{2x}=\lim\limits_{x\to0}\dfrac{9e^{3x}}{2}=\dfrac{9}{2}$

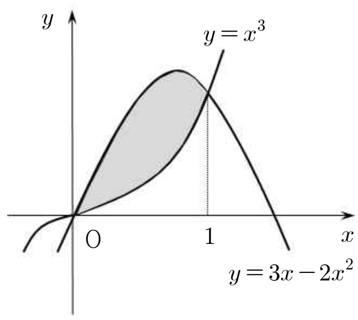
2. $f(2)=0,\ f'(x)=\sqrt{1+x^3}$ 이고 $f^{-1}(0)=2,\ f'(2)=3$ 이므로

$(f^{-1})'(x)=\dfrac{1}{f'(f^{-1}(x))}$ 에서 $(f^{-1})'(0)=\dfrac{1}{f'(f^{-1}(0))}=\dfrac{1}{f'(2)}=\dfrac{1}{3}$

3. $\dfrac{\tan x}{1+x^2}$ 은 기함수이므로 $\displaystyle\int_{-2}^{2}\frac{\tan x}{1+x^2}\,dx=0$

$$\int_{-2}^{2}\left(3+\sqrt{4-x^2}+\frac{\tan x}{1+x^2}\right)dx=\int_{-2}^{2}3\,dx+\int_{-2}^{2}\sqrt{4-x^2}\,dx=12+2\pi$$

4. $x^3=3x-2x^2,\ x(x^2+2x-3)=x(x+3)(x-1)=0$ 에서 $x=0,\ -3,\ 1$ 이므로 부피를 원통껍질 방법을 이용하여 구하면

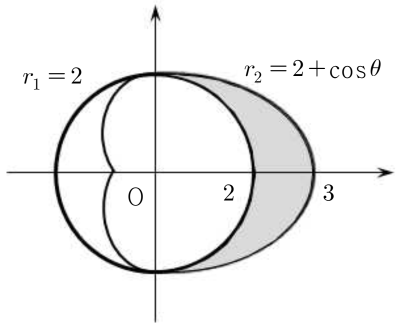


$$V=2\pi\int_0^1 x(3x-2x^2-x^3)dx$$

$$=2\pi\left[x^3-\frac{1}{2}x^4-\frac{x^5}{5}\right]_0^1=\frac{3}{5}\pi$$

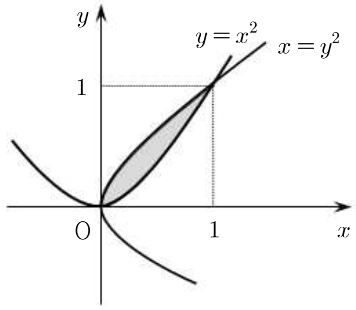
5. $2+\cos\theta=2$에서 $\theta=\dfrac{\pi}{2},\ \dfrac{3}{2}\pi$이므로 공통부분의 넓이 S는

$$S=2\int_0^{\pi/2}\frac{1}{2}[(2+\cos\theta)^2-2^2]\,d\theta$$
$$=\int_0^{\pi/2}(4\cos\theta+\cos^2\theta)d\theta$$
$$=\int_0^{\pi/2}\left(4\cos\theta+\frac{1+\cos2\theta}{2}\right)d\theta$$
$$=\left[4\sin\theta+\frac{1}{2}\theta+\frac{1}{4}\sin2\theta\right]_0^{\pi/2}=4+\frac{\pi}{4}$$



6. $V=\displaystyle\int_0^1\int_{x^2}^{\sqrt{x}}xy\,dy\,dx$

$$=\int_0^1\left[\frac{x}{2}y^2\right]_{x^2}^{\sqrt{x}}dx$$
$$=\int_0^1\frac{1}{2}(x^2-x^5)\,dx=\frac{1}{12}$$



7. $a=\langle 1,0,2\rangle,\ b=\langle 2,-1,4\rangle,\ a\cdot b=10,\ |a|=\sqrt{5}$이므로

$$\mathrm{Proj}_a b=\frac{a\cdot b}{|a|^2}\,a=\frac{10}{5}<1,0,2>=<2,0,4>$$

8. $u=\overrightarrow{AB}=<2,-4,4>,\ v=\overrightarrow{AC}=<4,-1,-2>,\ w=\overrightarrow{AD}=<t,3,-6>$라 할 때, 세 벡터 u,v,w가 같은 평면에 있으므로

$$u\cdot(v\times w)=\begin{vmatrix}2&-4&4\\4&-1&-2\\t&3&-6\end{vmatrix}=12t-24=0,\ t=2$$

9. $r'(t)=\langle 3t^{1/2},-4\sin4t,4\cos4t\rangle,\ |r'(t)|=\sqrt{9t+16}$이므로 곡선의 길이 L은

$$L=\int_0^1|r'(t)|\,dt=\int_0^1\sqrt{9t+16}\,dt$$
$$=\frac{1}{9}\int_{16}^{25}\sqrt{u}\,du=\frac{1}{9}\cdot\frac{2}{3}\left[u^{3/2}\right]_{16}^{25}=\frac{122}{27}$$

10. $(1,1,1)$은 $t=1,\ r(t)=\langle 1,t,t^2\rangle,\ r'(t)=<0,1,2t>,\ r''(t)=\langle 0,0,2\rangle$
$r'(1)=<0,1,2>,\ |r'(1)|=\sqrt{5},\ r''(1)=<0,0,2>,\ r'(1)\times r''(1)=<2,0,0>$
이므로 구하는 곡률은 $\dfrac{|r'(1)\times r''(1)|}{|r'(1)|^3}=\dfrac{2}{5\sqrt{5}}$

11. $(0,3,1)$은 $t=1$, $r(t)=\langle \ln t,\, 3t,\, t^3 \rangle$일 때 법평면의 법선벡터는 $r'(1)$. 이때 $r'(t)=\left\langle \frac{1}{t},\, 3,\, 3t^2 \right\rangle$에서 $r'(1)=<1,3,3>$이므로 법평면의 방정식은

$$1(x-0)+3(y-3)+3(z-1)=0,\quad x+3y+3z=12$$

12. $u=0,\ v=\frac{\pi}{2}$ 일 때 $x=0,\ y=\frac{\pi}{2}$이므로

$$\begin{aligned}\frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u}\\ &= \frac{3}{3x+2y}\sin v+\frac{2}{3x+2y}(-v\sin u)=\frac{3}{\pi}\end{aligned}$$

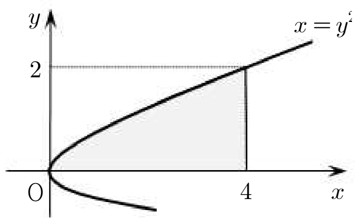
13. $f(x,y)=x^2+xy^3$, $\nabla f(x,y)=<2x+y^3,\ 3xy^2>$, $\nabla f(2,1)=<5,6>$

v방향의 단위벡터는 $u=\frac{1}{5}<3,4>$이므로 v방향의 f의 방향도함수는

$$D_u f(2,1)=\nabla f(2,1)\cdot u=<5,6>\cdot\frac{1}{5}<3,4>=\frac{39}{5}$$

14. $\displaystyle\int_0^2\int_{y^2}^4 f(x,y)\,dx\,dy$

$\displaystyle=\int_0^4\int_0^{\sqrt{x}} f(x,y)\,dy\,dx$



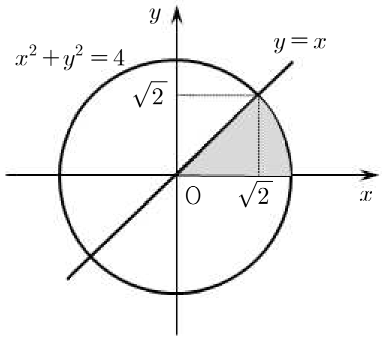
15. $\displaystyle\int_0^{\sqrt{2}}\int_y^{\sqrt{4-y^2}} e^{-x^2-y^2}\,dx\,dy$

$\displaystyle=\int_0^{\pi/4}\int_0^2 e^{-r^2}\,r\,dr\,d\theta$

$\displaystyle=\int_0^2 e^{-r^2}r\,dr\cdot\int_0^{\pi/4}d\theta$

$\displaystyle=-\frac{1}{2}\left[e^{-r^2}\right]_0^2\ \left[\theta\right]_0^{\frac{\pi}{4}}$

$\displaystyle=\frac{\pi}{8}(1-e^{-4})$



16. 완전미분방정식이 되기 위해서는 $\frac{\partial}{\partial y}(x^2+2xy+ky^2)=\frac{\partial}{\partial x}(x^2+2xy-1)$을 만족시켜야 하므로 $2x+2ky=2x+2y$에서 $k=1$

17. $y' - y = e^t y^2$ 에서 $w = y^{-1}$ 라 놓으면, $\dfrac{dw}{dt} + w = -e^t$

적분인자는 $e^{\int 1\,dt} = e^t$ 이므로 $\dfrac{d}{dt}[e^t w] = -e^{2t}$, $e^t w = -\dfrac{1}{2}e^{2t} + C$

$w = -\dfrac{1}{2}e^t + Ce^{-t}$, $y^{-1} = -\dfrac{1}{2}e^t + Ce^{-t}$

$y(0) = 1$에서 $C = \dfrac{3}{2}$. $y^{-1} = -\dfrac{1}{2}e^t + \dfrac{3}{2}e^{-t}$ 이므로

$$y(\ln 2) = \frac{1}{-\frac{1}{2}e^{\ln 2} + \frac{3}{2}e^{-\ln 2}} = -4$$

18. 적분인자는 $e^{\int \frac{1}{1+t}dt} = 1 + t$ 이므로 $\dfrac{d}{dt}[(t+1)y(t)] = \ln t$

$(t+1)y(t) = t\ln t - t + C$, $y(t) = \dfrac{t\ln t}{t+1} - \dfrac{t}{t+1} + \dfrac{C}{t+1}$

$y(1) = \dfrac{3}{2}$에서 $C = 4$ 이므로 $y(t) = \dfrac{t\ln t}{t+1} - \dfrac{t}{t+1} + \dfrac{4}{t+1}$, $y(e) = \dfrac{4}{e+1}$

19. $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - \dfrac{1}{4}(1 - 4x^2)y = 0$ 에서 $\dfrac{d^2y}{dx^2} + \dfrac{1}{x}\dfrac{dy}{dx} - \dfrac{1}{4}\left(\dfrac{1}{x^2} - 4\right)y = 0$,

$p(x) = \dfrac{1}{x}$ 라 하면, y_1과 일차독립인 해는

$$\begin{aligned}
y_1\int \frac{1}{{y_1}^2}\exp\left(-\int p(x)dx\right)dx &= \frac{\sin x}{\sqrt{x}}\int \frac{x}{\sin^2 x}\, e^{-\int \frac{1}{x}dx}\, dx \\
&= \frac{\sin x}{\sqrt{x}}\int \frac{x}{\sin^2 x}\, e^{-\ln x}\, dx \\
&= \frac{\sin x}{\sqrt{x}}\int \frac{x}{\sin^2 x}\, \frac{1}{x}dx \\
&= \frac{\sin x}{\sqrt{x}}\int \csc^2 x\, dx \\
&= \frac{\sin x}{\sqrt{x}}(-\cot x) \\
&= -\frac{\cos x}{\sqrt{x}} = -x^{-1/2}\cos x
\end{aligned}$$

따라서 $y_2 = x^{-1/2}\cos x$

20. $y = x^m$ 이라 할 때, 특성방정식은 $m^2 + 2m + 5 = 0$, $m = -1 \pm 2i$ 이므로
일반해는 $y = x^{-1}[A\cos(2\ln x) + B\sin(2\ln x)]$. 따라서 $m = -1$, $f(x) = 2\ln x$
이므로 $m + f(e) = -1 + 2\ln e = 1$

21. $y_1 = e^{-2x}\cos x$, $y_2 = e^{-2x}\sin x$가 2계 상수계수 상미분방정식 $y'' + ay' + by = 0$의 해이므로 $y = e^{\lambda x}$라 할 때, 특성방정식은 $\lambda^2 + a\lambda + b = 0$이고 해가 $\lambda = -2 \pm i$이므로
$a = 4$, $b = 5$. 따라서 $a + b = 9$

22. $g(t) = \begin{cases} 2, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \\ 3, & t \ge 2 \end{cases}$

$$\begin{aligned} g(t) &= 2[1 - u(t-1)] + 0[u(t-1) - u(t-2)] + 3u(t-2) \\ &= 2 - 2u(t-1) + 3u(t-2) \end{aligned}$$

23. $f(t) = t\cos(2t)$일 때

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t\cos(2t)\} = -\frac{d}{ds}\mathcal{L}\{\cos(2t)\} = -\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = \frac{s^2-4}{(s^2+4)^2}$$

$\mathcal{L}\{f(t)\} = \int_0^\infty t\cos(2t)e^{-st}dt$ 이므로 $\int_0^\infty t\cos(2t)e^{-st}dt = \dfrac{s^2-4}{(s^2+4)^2}$

$s = 3$일 때 $\int_0^\infty t\cos(2t)e^{-3t}dt = \dfrac{9-4}{(9+4)^2} = \dfrac{5}{169}$

24. $F(s) = \dfrac{4e^{-\pi s}}{1+s^2}$, $\mathcal{L}^{-1}\{F(s)\} = f(t)$라 하면

$\mathcal{L}^{-1}\left\{\dfrac{4e^{-\pi s}}{1+s^2}\right\} = 4\mathcal{L}^{-1}\left\{\dfrac{1}{1+s^2}e^{-\pi s}\right\} = 4\sin(t-\pi)u(t-\pi)$이므로

$$f(t) = \begin{cases} 0, & 0 \le t < \pi \\ 4\sin(t-\pi), & t \ge \pi \end{cases}$$

따라서 $f\left(\dfrac{3\pi}{2}\right) = 4$

25. $y'(t) + e^{-2t} * y(t) = 1$이므로 $\mathcal{L}\{y(t)\} = Y(s)$라 하면
$sY(s) - 1 + \dfrac{1}{s+2}Y(s) = \dfrac{1}{s}$, $\left(s + \dfrac{1}{s+2}\right)Y(s) = 1 + \dfrac{1}{s}$,

$Y(s) = \dfrac{s+2}{s^2+s} = \dfrac{2}{s} - \dfrac{1}{s+1}$ 에서 $y(t) = 2 - e^{-t}$