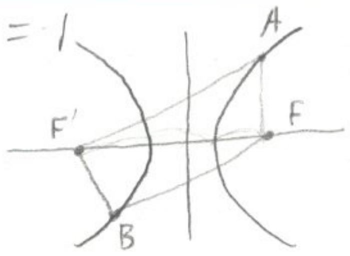
문제 2번 (반드시 해당문제와 일치하여야 함)

1. $\frac{x^2}{5} - \frac{y^2}{27} = \frac{1}{2}$ $\quad \frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{27}{2}} = 1$

$F(4,0),\ F'(-4,0)$



$\overline{AF} = k,\ \overline{AF'} = k+\sqrt{10}$

$\overline{FF'} = 8$

$\cos(\angle FAF') = \frac{7}{25}$

$\frac{7}{25} = \frac{k^2+(k+\sqrt{10})^2-8^2}{2\cdot k\cdot(k+\sqrt{10})}$ $\quad 2k^2+2\sqrt{10}k-75=0$

$k = \frac{3\sqrt{10}}{2}$ $\quad (k>0)$

$A(a,b)$ $\quad \frac{a^2}{5} - \frac{b^2}{27} = \frac{1}{2}$

$\sqrt{(a-4)^2+b^2} = \frac{3\sqrt{10}}{2}$ $\quad b^2 = \frac{27}{5}a^2 - \frac{27}{2}$

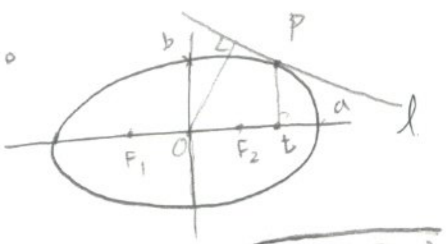
$32a^2-40a-100=0$ $\quad (a>0)$ $\quad a = \frac{5}{2}$

$b = \frac{9}{2}$

$\cos(\angle FAF') = \cos(\angle F'BF)$ A와 B는 원점 대칭

$A\left(\frac{5}{2}, \frac{9}{2}\right)$ $\quad B\left(-\frac{5}{2}, -\frac{9}{2}\right)$

2.



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad b = \frac{\sqrt{3}}{2}a$

$\frac{x^2}{a^2} + \frac{4y^2}{3a^2} = 1$ $\quad a>b$

$P\left(t, \sqrt{\frac{3}{4}a^2 - \frac{3}{4}t^2}\right)$

$\frac{x^2}{a^2} + \frac{4y^2}{3a^2} - 1 = 0$을 x에 대해 미분 $\quad \frac{2}{a^2}x + \frac{4}{3a^2}2y\cdot\frac{dy}{dx} = 0$

점 P에서 $\frac{dy}{dx} = \frac{-3t}{4\sqrt{\frac{3}{4}a^2 - \frac{3}{4}t^2}}$

직선 l $\quad y = \frac{-3t}{4\sqrt{\frac{3}{4}a^2-\frac{3}{4}t^2}}(x-t) + \sqrt{\frac{3}{4}a^2 - \frac{3}{4}t^2}$

$0 = \frac{3}{4}tx + \sqrt{\frac{3}{4}a^2 - \frac{3}{4}t^2}\,y - \frac{3}{4}a^2$

$(0,0)$에서 l까지의 거리 제곱 $= \frac{3a^4}{4a^2-t^2} = f(t)$

$\frac{1}{a^3}\int_0^a f(t)\,dt = 3\int_0^a \frac{a}{4a^2-t^2}\,dt = \frac{3}{4}\int_0^a \frac{1}{2a-t} + \frac{1}{2a+t}\,dt$

$= \frac{3}{4}\left[-\ln(2a-t) + \ln(2a+t)\right]_0^a = \frac{3}{4}\ln 3$

3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$P\left(t, \sqrt{1-\frac{t^2}{a^2}}\cdot b\right)$

$\frac{2}{a^2}x + \frac{1}{b^2}\cdot 2y\cdot\frac{dy}{dx} = 0$

점 P에서 미분계수 $= \frac{-\frac{b}{a^2}t}{\sqrt{1-\frac{t^2}{a^2}}}$

직선 l $\quad y = \frac{-\frac{b}{a^2}t}{\sqrt{1-\frac{t^2}{a^2}}}(x-t) + \sqrt{1-\frac{t^2}{a^2}}\cdot b$

$f(t) = \frac{a^4b^2}{b^2t^2 - a^2t^2 + a^4}$

$F_1(-\sqrt{a^2-b^2}, 0)$ $\quad F_2(\sqrt{a^2-b^2}, 0)$

$\overline{PF_1} = \sqrt{(t+\sqrt{a^2-b^2})^2 + \left(\sqrt{1-\frac{t^2}{a^2}}\cdot b\right)^2}$

$\overline{PF_2} = \sqrt{(t-\sqrt{a^2-b^2})^2 + \left(\sqrt{1-\frac{t^2}{a^2}}\times b\right)^2}$

$h(t) = \overline{PF_1}\times\overline{PF_2} = \frac{b^2}{a^2}t^2 - t^2 + a^2 = \frac{1}{a^2}(b^2t^2 - a^2t^2 + a^4)$

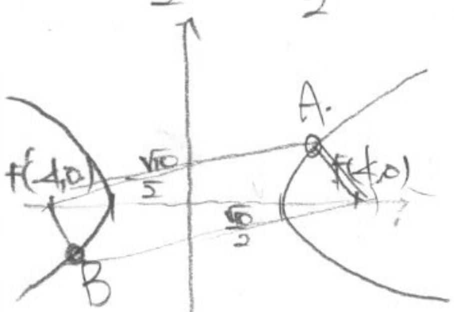
$f(t)\times h(t) = a^2b^2$

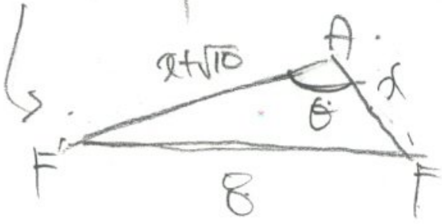
이 줄 위에 답안 작성 시 무효 처리됨

문제 2번 (반드시 해당문제와 일치하여야 함)

2.(1) $\frac{x^2}{5} - \frac{y^2}{27} = \frac{1}{2}$

$\Rightarrow \frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{27}{2}} = 1$, 초점 $F(4,0)$ $F'(-4,0)$





$\overline{AF} = x$ 라고 할때

쌍곡선 성질에 의해 $\overline{AF'} = x+\sqrt{10}$

$\cos\theta = \frac{7}{25}$ 이므로 코사인 제 2법칙을 쓰면

$64 = x^2 + (x+\sqrt{10})^2 - 2x(x+\sqrt{10})\cos\theta$

$\Rightarrow 4x^2 + 4\sqrt{10}x - 150 = 0$

$(2x - 3\sqrt{10})(2x + 5\sqrt{10}) = 0$

$x = \frac{3\sqrt{10}}{2}$ ($\because x > 0$ 이므로)

$A(\alpha, \beta)$ 라고 할때

$(\alpha-4)^2 + \beta^2 = \frac{3\sqrt{10}}{2}$

$\frac{\alpha^2}{5} - \frac{\beta^2}{27} = \frac{1}{2}$ 이므로 둘을 연립해보면

$\alpha = \frac{5}{2}$, $\beta = \frac{9}{2}$ 가 나온다. $A\left(\frac{5}{2}, \frac{9}{2}\right)$.

이때 쌍곡선은 원점에 점대칭이므로

$\boxed{A\left(\frac{5}{2}, \frac{9}{2}\right),\ B\left(-\frac{5}{2}, -\frac{9}{2}\right)}$ 가 된다.

2.(2) $b = \frac{\sqrt{3}}{2}a$ 이므로 $\frac{x^2}{a^2} + \frac{y^2}{\frac{3}{4}a^2} = 1$ 이고

$\frac{2x}{a^2}dx + \frac{2y}{\frac{3}{4}a^2}dy = 0$ 이므로 $\frac{dy}{dx} = -\frac{3x}{4y}$ 가 된다

$P(t, s)$ 라고 할때 $\Rightarrow 3t^2 + 4s^2 = 3a^2$.

$l: y = -\frac{3t}{4s}x + \frac{3t^2}{4s} + s$

$\Rightarrow 3tx + 4sy = 3t^2 + 4s^2$

$f(t) = \left(\frac{3t^2+4s^2}{\sqrt{9t^2+16s^2}}\right)^2 = \frac{9a^4}{9t^2+16s^2}$

$s^2 = \frac{3}{4}(a^2 - t^2)$

$= \frac{3a^4}{4a^2 - t^2}$ 이 된다

$\frac{1}{a^3}\int_0^a f(t)\,dt = \frac{1}{a^3}\int_0^a \frac{3a^4}{4a^2-t^2}\,dt$

$= 3a\int_0^a \frac{1}{4a(2a-t)} + \frac{1}{4a(2a+t)}\,dt$

$= \frac{3}{4}\left[-\ln(2a-t) + \ln(2a+t)\right]_0^a$

$= \frac{3}{4}\ln 3$. $\boxed{\frac{3}{4}\ln 3}$

2.(3) 위에서 알수 있듯이 $P(t, s)$ 라 할때

$l: y = \frac{-tb^2}{sa^2}x + \frac{t^2b^2}{sa^2} + s$

$tb^2x + sa^2y = t^2b^2 + s^2a^2$

$f(t) = \left(\frac{t^2b^2+s^2a^2}{\sqrt{t^2b^4+s^2a^4}}\right)^2 = \frac{a^4b^4}{t^2b^4+s^2a^4}$ 이 된다.

$h(t) = \overline{PF_1} \times \overline{PF_2}$

$= \sqrt{(t-\sqrt{a^2-b^2})^2 + s^2} \times \sqrt{(t+\sqrt{a^2-b^2})^2 + s^2}$ ($\because F_1(\sqrt{a^2-b^2}, 0)$ $F_2(-\sqrt{a^2-b^2}, 0)$ 이므로)

$= \sqrt{(t^2 + (a^2-b^2) + s^2)^2 - 4t^2(a^2-b^2)}$ ($s^2 = b^2 - \frac{b^2}{a^2}t^2$ 이므로)

$= \sqrt{\left(-t^2 + \frac{b^2}{a^2}t^2 + a^2\right)^2}$

$= \frac{1}{a^2}\left(a^4 - t^2(a^2-b^2)\right)$ 이 된다.

$f(t) \times h(t) = \frac{a^4b^4}{t^2b^4+s^2a^4} \times \frac{1}{a^2}\left(a^4 - t^2(a^2-b^2)\right)$ $s^2 = b^2 - \frac{b^2}{a^2}t^2$

$= \frac{a^4b^4}{b^2\left(a^4 - t^2(a^2-b^2)\right)} \times \frac{1}{a^2}\left(a^4 - t^2(a^2-b^2)\right)$

$= a^2b^2$ $\boxed{a^2b^2}$

이 줄 아래 답안 작성 시 무효 처리됨