Problem 1-4

Model the following problems to the matroid intersection instance, and provide a proof.

Problem 1

You are given N bags, and each bag is consisted of M vectors of R^k. Determine if there is a way to pick exactly 1 vector from each bag (thus N vectors in total), such that the chosen vectors are linearly independent.

Optional. Implement the above algorithm, and get Accepted verdict in this link.

Problem 2

Given a graph where every edge have an integer color, find a spanning tree where every edge in spanning tree have different colors.

Optional. Implement the above algorithm, and get Accepted verdict in this link.

Problem 3

You have a graph where edges have weights and color *red, green, blue*. Some of your friends can't see the red edge, and other friends can't see the blue edge. Currently, the graph is connected for both friends. Find a minimum cost edge subset that looks connected for both group of your friends.

Optional. Implement the above algorithm, and get Accepted verdict in this link.

Problem 4

You are given an connected **bipartite graph** $G = (L, R, E)$ $(V(G) = L \cup R, L \cap R = \emptyset)$ with at least 2 vertices. A vertex is called a leaf if it have degree 1. Find a spanning tree, where every leaf belongs to R.

Optional. Implement the above algorithm, and get Accepted verdict in this link.
Solution. Click here

Problem 5

Konig's theorem states that the size of minimum vertex cover and maximum matching is same in bipartite graph. Prove Konig's theorem with Matroid Intersection Theorem.

Problem 6

Problem 6A. Prove that matroid M has k disjoint bases $\iff kr(X) + |E - X| \geq kr(E)$ for all $X \subseteq E$.

Problem 6B. Prove that G has k edge disjoint spanning trees $\iff$ for every partition of $V(G) = P_1 \cup \ldots, \cup P_s$, the number of edges having ends in distinct P_i (call this $C(P)$) is at least $(s-1)k$. This result is known as *Nash-William's Theorem.*

Hint: Use formula from 6A. Which X should we take?

Problem 6C (Optional). Show that any $2k$-connected graph has k edge disjoint spanning trees.