

TABLE OF INTEGRALS

BASIC FORMS

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| 1. $\int u \, dv = uv - \int v \, du$ | 11. $\int \csc u \cot u \, du = -\csc u + C$ |
| 2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$ | 12. $\int \tan u \, du = \ln \sec u + C$ |
| 3. $\int \frac{du}{u} = \ln u + C$ | 13. $\int \cot u \, du = \ln \sin u + C$ |
| 4. $\int e^u \, du = e^u + C$ | 14. $\int \sec u \, du = \ln \sec u + \tan u + C$ |
| 5. $\int a^u \, du = \frac{a^u}{\ln a} + C$ | 15. $\int \csc u \, du = \ln \csc u - \cot u + C$ |
| 6. $\int \sin u \, du = -\cos u + C$ | 16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$ |
| 7. $\int \cos u \, du = \sin u + C$ | 17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$ |
| 8. $\int \sec^2 u \, du = \tan u + C$ | 18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$ |
| 9. $\int \csc^2 u \, du = -\cot u + C$ | 19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left \frac{u+a}{u-a} \right + C$ |
| 10. $\int \sec u \tan u \, du = \sec u + C$ | 20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left \frac{u-a}{u+a} \right + C$ |

FORMS INVOLVING $\sqrt{a^2 + u^2}, a > 0$

21. $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
22. $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
23. $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$
24. $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$
25. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$
26. $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
27. $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
28. $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29. $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$



FORMS INVOLVING $\sqrt{a^2 - u^2}$, $a > 0$

- 30. $\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$
- 31. $\int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$
- 32. $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$
- 33. $\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$
- 34. $\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$
- 35. $\int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$
- 36. $\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$
- 37. $\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$
- 38. $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$

FORMS INVOLVING $\sqrt{u^2 - a^2}$, $a > 0$

- 39. $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$
- 40. $\int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$
- 41. $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$
- 42. $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$
- 43. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$
- 44. $\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$
- 45. $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$
- 46. $\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$

 **TABLE OF INTEGRALS**

FORMS INVOLVING $a + bu$

47. $\int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$
48. $\int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$
49. $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$
50. $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
51. $\int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$
52. $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
53. $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$
54. $\int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$
55. $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$
56. $\int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$
57. $\int \frac{du}{u \sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \quad \text{if } a > 0$
 $= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \quad \text{if } a < 0$
58. $\int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u \sqrt{a + bu}}$
59. $\int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u \sqrt{a + bu}}$
60. $\int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$
61. $\int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$
62. $\int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$



TRIGONOMETRIC FORMS

63. $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4} \sin 2u + C$

64. $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4} \sin 2u + C$

65. $\int \tan^2 u \, du = \tan u - u + C$

66. $\int \cot^2 u \, du = -\cot u - u + C$

67. $\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$

68. $\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$

69. $\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$

70. $\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + C$

71. $\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$

72. $\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$

73. $\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$

74. $\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$

75. $\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$

76. $\int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$

77. $\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$

78. $\int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$

79. $\int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$

80. $\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$

81. $\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\int u \sin u \, du = \sin u - u \cos u + C$

83. $\int u \cos u \, du = \cos u + u \sin u + C$

84. $\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$

85. $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$

86. $\int \sin^n u \cos^m u \, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du$
 $= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u \, du$

INVERSE TRIGONOMETRIC FORMS

87. $\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$

88. $\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$

89. $\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$

90. $\int u \sin^{-1} u \, du = \frac{2u^2-1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

91. $\int u \cos^{-1} u \, du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$

92. $\int u \tan^{-1} u \, du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$

93. $\int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$

94. $\int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$

95. $\int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2} \right], \quad n \neq -1$

TABLE OF INTEGRALS

EXPONENTIAL AND LOGARITHMIC FORMS

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| 96. $\int ue^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$ | 100. $\int \ln u du = u \ln u - u + C$ |
| 97. $\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$ | 101. $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$ |
| 98. $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$ | 102. $\int \frac{1}{u \ln u} du = \ln \ln u + C$ |
| 99. $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$ | |

HYPERBOLIC FORMS

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| 103. $\int \sinh u du = \cosh u + C$ | 108. $\int \operatorname{csch} u du = \ln \tanh \frac{1}{2} u + C$ |
| 104. $\int \cosh u du = \sinh u + C$ | 109. $\int \operatorname{sech}^2 u du = \tanh u + C$ |
| 105. $\int \tanh u du = \ln \cosh u + C$ | 110. $\int \operatorname{csch}^2 u du = -\operatorname{coth} u + C$ |
| 106. $\int \operatorname{coth} u du = \ln \sinh u + C$ | 111. $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$ |
| 107. $\int \operatorname{sech} u du = \tan^{-1} \sinh u + C$ | 112. $\int \operatorname{csch} u \operatorname{coth} u du = -\operatorname{csch} u + C$ |

FORMS INVOLVING $\sqrt{2au - u^2}$, $a > 0$

113. $\int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$
114. $\int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$
115. $\int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$
116. $\int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left(\frac{a-u}{a} \right) + C$
117. $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1} \left(\frac{a-u}{a} \right) + C$
118. $\int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$
119. $\int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u+3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$
120. $\int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$