1-1-(1)

$\boxed{5}-\boxed{7}-\boxed{9}-\boxed{11}-\boxed{13}$ 이 <조건>을 만족시킨다.

$\frac{5+9}{2}=7,\ \frac{7+11}{2}=9,\ \frac{9+13}{2}=11$ 이기 때문이다.

1-1-(2)

$\frac{0+A_2}{2}=A_1,\ \frac{A_1+A_3}{2}=A_2,\ \ldots\ \frac{A_i+A_{i+2}}{2}=A_{i+1}\ (1\le i\le 47)\ \ldots\ \frac{A_{98}+2020}{2}=A_{99}$ 이다.

$A_i+A_{i+2}=2A_{i+1}$

$A_{i+2}-A_{i+1}=A_{i+1}-A_i$ 이를 통해 A_i가 등차수열 임을 알 수 있다.

~~$A_i = A_1+(i-1)d$ 라고 하자~~

~~$A_1+(i-1)d$ 라고 하자~~

~~$A_i = A_1+$~~

$A_n=A_1+(n-1)d$ 라고 하자 $(1\le n\le 99)$

$A_2=A_1+d$ $\quad \frac{0+A_1+d}{2}=A_1$ $\quad A_1=d$

$A_{99}=A_1+98d$ $\quad = 2020-99d$

$A_{98}=A_1+97d$ $\quad \frac{2020+A_1+97d}{2}=A_1+98d$ $\quad d=\frac{101}{5},\ A_1=\frac{101}{5}$

$A_n=\frac{101}{5}n$ $\quad \therefore A_{45}=909$

~~1-1-(3)~~

1-1-(3)

$\frac{\ln B_{i-1}+\ln B_{i+1}}{2}=\ln B_i\ (2\le i\le n-2)$

$\ln B_{i+1}-\ln B_i=\ln B_i-\ln B_{i-1}$

따라서 $\ln B_i$도 등차수열이다.

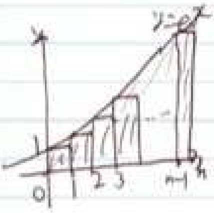
$\ln B_k=\ln B_1+(k-1)d'$ $\quad \ln B_{n-2}=\ln B_1+(n-3)d'$

$\ln B_2=\ln B_1+d'$ $\quad 2\ln B_{n-1}=\ln B_1+(n-3)d'+n$

$\ln B_1+d'=2\ln B_1$ $\quad \ln B_1=d'$ $\quad 2\ln B_{n-1}=(n-2)\ln B_1+n$

$2\{(n-1)\ln B_1\}=(n-2)\ln B_1+n$

$\therefore \ln B_k=k$ $\quad \therefore B_k=e^k\ (1\le k\le n-1)$ $\quad n\ln B_1=n$ $\quad B_1=e$ $\quad d'=1$



그래프를 통해 $B=\sum_{k=0}^{n-1}e^k<\int_0^n e^x\,dx$ 임을 알 수 있다.

~~$\ln B\le \ln(e^n-1)<n$~~ 이다. $\quad \therefore \ln B<n$

$B<e^n-1$

$\ln B<\ln(e^n-1)<\ln e^n=n$

1-2-(1) 석판 모서리 부분 칸에 들어갈 수 H 를 구하자.

$H=\frac{\left\{\sum_{i=1}^{n}4i^3+4i+10^4\right\}+1}{n+1}$

$H=\frac{n^2(n+1)^2+2n(n+1)+10^4n+1}{n+1}\le 10^4$

$=$ ~~$n^2(n+1)+2n+\frac{10^4+1}{n+1}$~~

$n^2(n+1)^2+2n(n+1)+1\le 10^4$

$n^2(n+1)^2+2n(n+1)+1-10^4\le 0$

$(n(n+1)+1-10^2)(n(n+1)+1+10^2)\le 0$

$-1-10^2<0\le n(n+1)\le 99$

$n^2+n-99\le 0$

$1\le n\le \frac{\sqrt{397}-1}{2}$

이를 만족하는 자연수 중 최대 n은 9이다.

1-2-(2) $A+1=2x_1$, $A+2020=2x_2$, $A+x_2=2x_1$

$2019+x_2=3A$ (∵ 제시문 나))

$x_{i-1}+x_{i+1}=2x_i$ $\quad x_i$는 등차수열이다.

$x_{i+1}-x_i=x_i-x_{i-1}$ $\quad (2\le i\le 2018)$

$x_k=x_1+(k-1)d''$ $\quad A+x_2=2x_1,\ x_{2018}+2020=2x_{2019}$ $\quad d''=1-\frac{A}{2020}$

$A+x_1+d''=2x_1$ $\quad$ ~~2020~~ $\quad x_1=1+\frac{2019}{2020}A$

$A+d''=x_1$ $\quad x_1+2019d''=2020$

$x_1-d''=A$

$2019+1+\frac{2019}{2020}A+1-\frac{A}{2020}=3A$

~~$\frac{4049}{2020}$~~ $\frac{2021}{1010}A=2021$

$\therefore A=1010$

2번 문항 (반드시 해당문항과 일치하여야 함)

2-1-(1) $y = x^3 + 16$

$g(x) = x^3 - kx + 16 \quad (k > 0)$

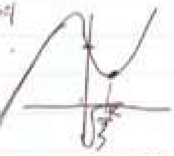
$h(x) = x^3 - (k+5)x + 16$ 이라고 하자.

$g(x)$는 극 양수인 근이 1개이다.

$g'(x) = 3x^2 - k$ 에서

$x = \sqrt{\frac{k}{3}}$ 일 때

$g(x) = 0$이다.



$g(x) = 0$이어야 하므로

$+\frac{2}{3}k\sqrt{\frac{k}{3}} = 16$

$\sqrt{\frac{k^3}{3}} = +24 \quad \sqrt{k} = +2\sqrt{3}$

$\sqrt{k^3} = -2\sqrt{3} \quad k = 12$

$\sqrt{\frac{k}{3}} = 2$ 에서 x좌표는 2이다.

$\therefore K = 12$

$h(x) = x^3 - 17x + 16$

$h(x) = (x-1)\left(x - \left(\frac{-1-\sqrt{65}}{2}\right)\right)\left(x - \left(\frac{-1+\sqrt{65}}{2}\right)\right)$

$h(x)$가 양의 실근을 2개 가진다.

Q, R의 x 좌표의 곱은 $\frac{\sqrt{65}-1}{2} \times 2 \times 1$

$= \sqrt{65} - 1$ 이다.

2-1-(2) $\tan\theta_t = \frac{t+1-t}{1+t(t+1)} = \frac{1}{t^2+t+1} \quad \left(0 < \theta_t < \frac{\pi}{2}\right)$

$1 + \tan^2\theta_t = \sec^2\theta_t$

$\frac{(t^2+t+1)^2 + 1}{(t^2+t+1)^2} = \sec^2\theta_t$

$\sec\theta_t = \frac{\sqrt{(t^2+t+1)^2+1}}{t^2+t+1}$

$\cos^2\theta_t = \frac{(t^2+t+1)^2}{(t^2+t+1)^2+1}$

$\sin^2\theta_t = 1 - \frac{(t^2+t+1)^2}{(t^2+t+1)^2+1} = \frac{1}{(t^2+t+1)^2+1}$

$\csc\theta_t = f(t) = \sqrt{(t^2+t+1)^2+1}$

$f(1) = \sqrt{10}, \quad f'(t) = \frac{\cancel{2}(t^2+t+1)(2t+1)}{\cancel{2}\sqrt{(t^2+t+1)^2+1}} \quad f'(1) = \frac{9}{10}\sqrt{10}$

2-1-(3) $(\sin x + \sin 2x)(\cos x + \cos 2x)$

$= \sin x\cos x + \sin x\cos 2x + \sin 2x\cos x + \sin 2x\cos 2x$

$= \frac{1}{2}\sin 2x + \sin 3x + \frac{1}{2}\sin 4x$

$\therefore C + \int_0^{\theta}(\sin x + \sin 2x)(\cos x + \cos 2x)\,dx$

$= C - \left(\frac{\cos 2\theta}{4} + \frac{\cos 3\theta}{3} + \frac{\cos 4\theta}{8} - \frac{17}{24}\right) \geq 0 \quad (\theta \geq 0)$

$-\frac{17}{24} + \frac{\cos 2x}{4} + \frac{\cos 3x}{3} + \frac{\cos 4x}{8} \leq C$

$\cos 2\theta, \cos 3\theta, \cos 4\theta$ 모두 최대가 되면

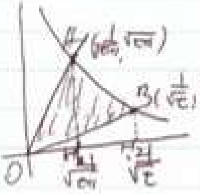
$\frac{\cos 2\theta}{4} + \frac{\cos 3\theta}{3} + \frac{\cos 4\theta}{8}$도 최대가 된다. $x = 0$이면 된다.

$\therefore \cancel{C \geq \frac{17}{24}}$

$\therefore C \geq 0$

C의 ~~최소값은~~ $\frac{17}{24}$ $\therefore C$의 최소값은 0이다.

2-2-(1)



$\triangle$ 삼각형 넓이를 S라고 하자

$\cancel{S = \frac{1}{2} + \int}$

$S = \frac{1}{2} + \int_{\frac{1}{\sqrt{t+1}}}^{\frac{1}{\sqrt{t}}} \frac{1}{x}\,dx - \frac{1}{2}$

$= \int_{\frac{1}{\sqrt{t+1}}}^{\frac{1}{\sqrt{t}}} \frac{1}{x}\,dx$

$= \frac{1}{2}\ln\frac{t+1}{t}$

$= A(t)$

$tA(t) = \frac{1}{2}t\ln\left(1+\frac{1}{t}\right)$

$= \frac{1}{2}\ln\left(1+\frac{1}{t}\right)^t$

$\therefore \lim_{t\to\infty} tA(t) = \lim_{t\to\infty}\frac{1}{2}\ln\left(1+\frac{1}{t}\right)^t$

$= \frac{1}{2}$

2-2-(2) $tx_t = -\ln\cos x_t$

$\left(0 < x_t < \frac{\pi}{2}\right)$

$\cos x_t = e^{-tx_t}$

$\sec x_t = e^{tx_t}$

$\tan x_t = \sqrt{e^{2tx_t} - 1}$

$\lim_{t\to\infty} x_t = \frac{\pi}{2}$ $\therefore \lim$

$= \lim_{t\to 0}\frac{-\ln\cos x_t}{t}$

$y = -\ln\cos x$

$dy = +\tan x\,dx$

$\frac{dy}{dx} = \tan x$

$s(t) = \int_0^{x_t}\sqrt{1+\tan^2 x}\,dx$

$= \int_0^{x_t}\sec x\,dx$

$= \int_0^{x_t}\frac{\sec x\tan x + \sec^2 x}{\tan x + \sec x}\,dx$

(∵ 제시문(다))

$= \left[\ln\left(\tan x + \sec x\right)\right]_0^{x_t}$

$= \ln(\tan x_t + \sec x_t)$

$= \ln(1 + \sin x_t) - \ln(\cos x_t)$

$\lim_{t\to 0}\frac{s(t)}{t} = \lim_{t\to 0}\frac{\ln(1+\sin x_t) - \ln(\cos x_t)}{t}$

$= \lim_{t\to 0}\frac{\ln(1+\sin x_t)}{t} + \lim_{t\to 0}\frac{-\ln(\cos x_t)}{t}$

$= \frac{\pi}{2} + 0$

$= \frac{\pi}{2}$

이 줄 아래는 답안 작성을 하지 말 것